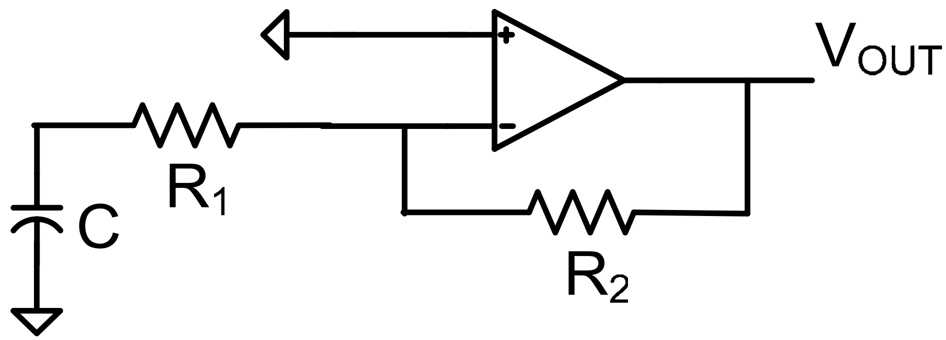
EE 230
Lecture 25

Waveform Generators

- Sinusoidal Oscillators

Quiz 17

Determine the characteristic equation of the following circuit.



And the number is ?

1 3 8

5 4

2 6

9 7

And the number is ?

1

3

8

5

4

3

2

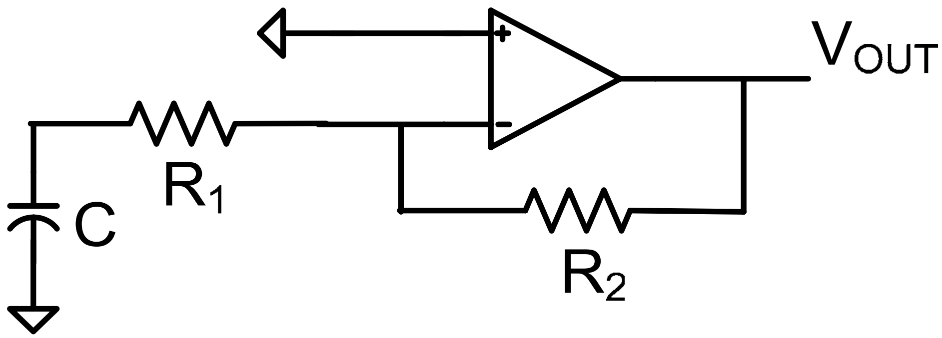
6

9

7

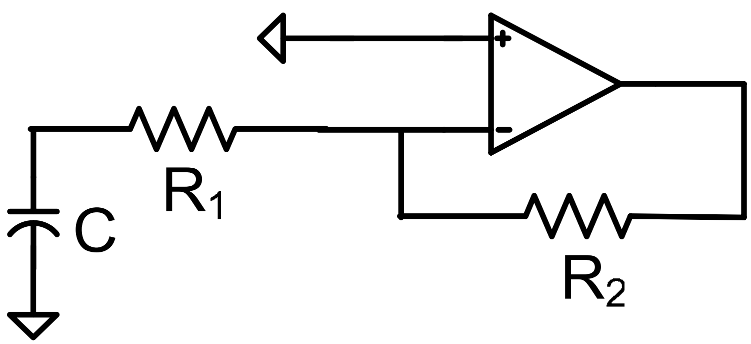
Quiz 17

Determine the characteristic equation of the following circuit.



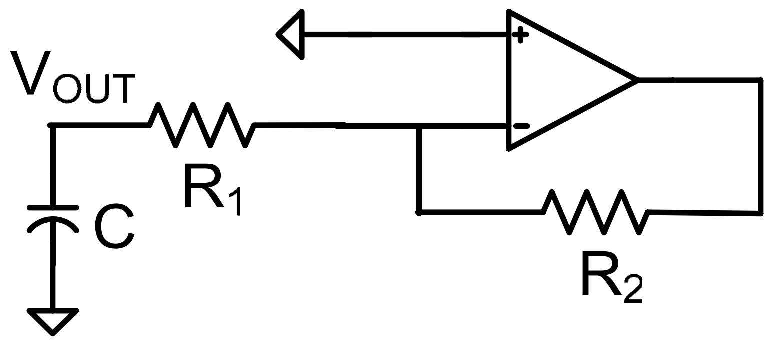
Solution:

For emphasis, draw the dead network as:



Quiz 17

Solution: Assign a voltage V_{OUT} to the node shown



By KCL

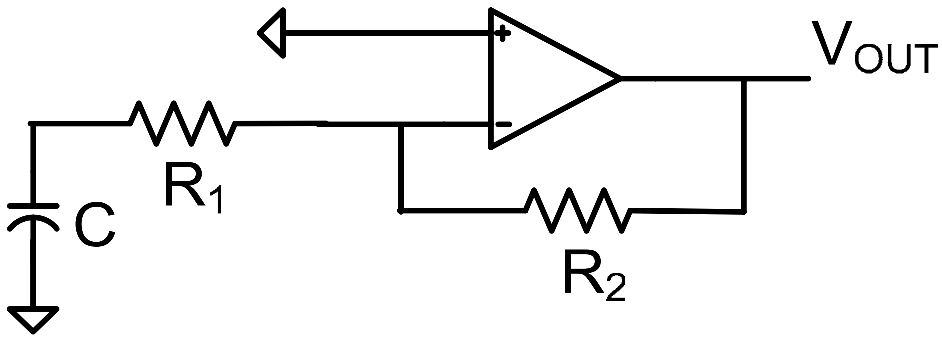
$$V_{OUT}\left(sC+\frac{1}{R_1}\right)=0$$

Thus

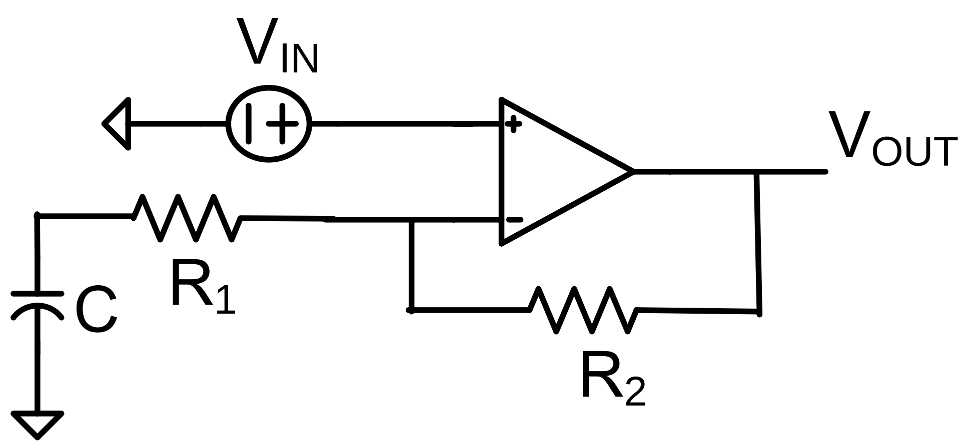
$$D(s)=sCR_1+1$$

Quiz 17

Determine the characteristic equation of the following circuit.

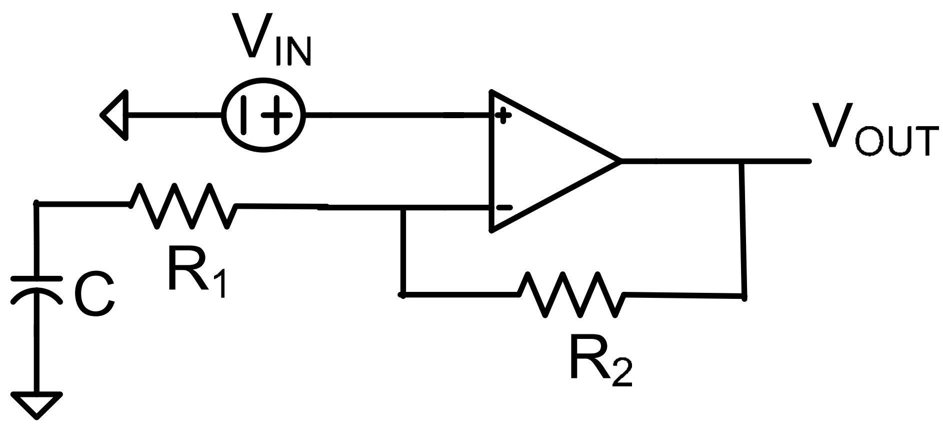


Solution: Alternately, assign an input without changing the dead network



Quiz 17

Solution:



$$T(s) = 1 + \frac{R_2}{R_1 + {}^{1}\!/_{sC}}$$

$$T(s) = \frac{1 + (R_1 + R_2)Cs}{sCR_1 + 1}$$

Thus

$$D(s) = sCR_1 + 1$$

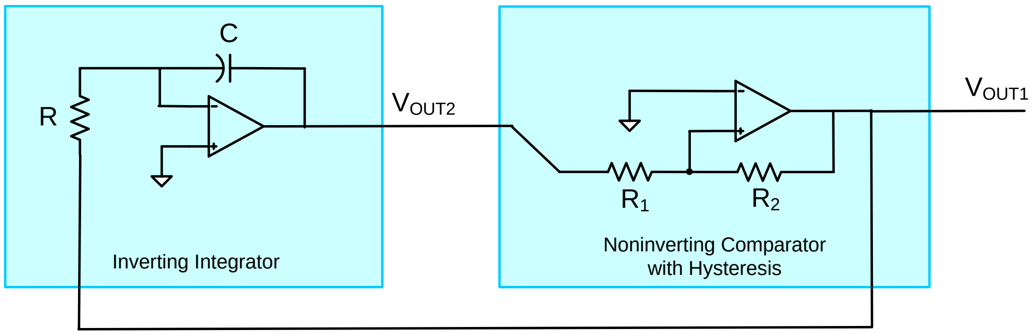
Review from Last Time:

Theorem: The characteristic polynomial $D(s)$ of a system can be obtained by assigning an output variable to the "dead network" and using circuit analysis techniques to obtain an expression that involves only the output variable expressed in the form $X_o F(s) = 0$. When expressed in this form, $F(s)$ when written in polynomial form is the characteristic polynomial of the system. i.e.

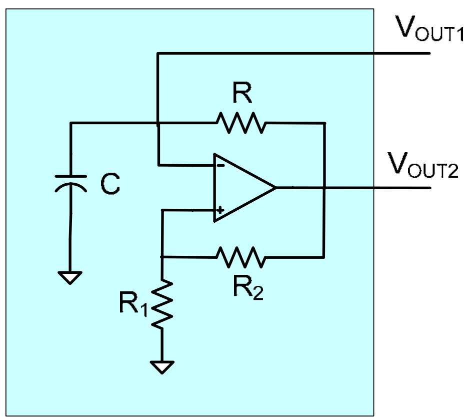
$$D(s) = F(s).$$

Review from Last Time:

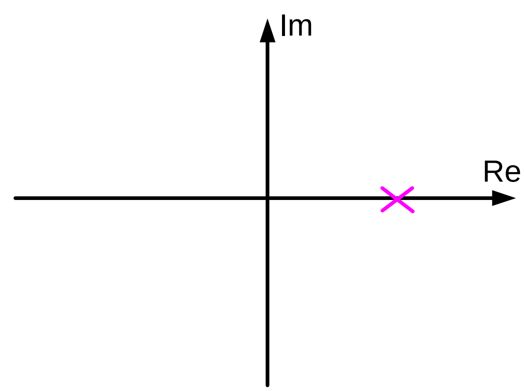
Pole Locations of Waveform Generators



$$p = \frac{1}{RC}\left(\frac{1-\theta}{\theta}\right)$$



$$p = \left(\frac{R_2}{R_1}\right)\frac{1}{RC}$$



Both have a single pole on the positive real axis

Sinusoidal Oscillators

- The previous two circuits provided square wave and triangular outputs
  - previous two circuits had a RHP pole on positive real axis

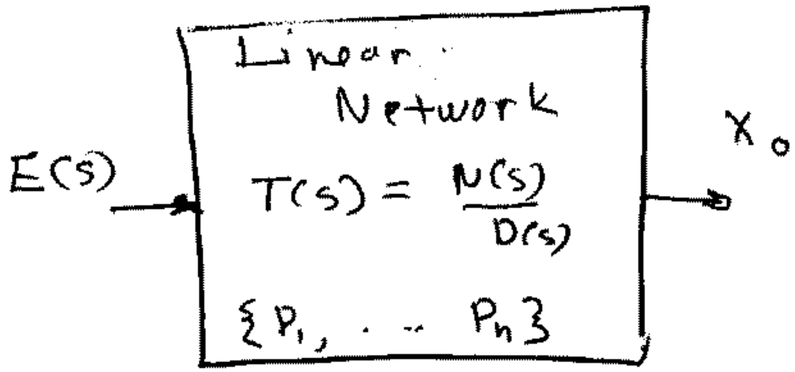
- What properties of a circuit are needed to provide a sinusoidal output

- What circuits have these properties

What properties of a circuit are needed to provide a sinusoidal output?

- Insight into how a sinusoidal oscillator works
- Barkhausen Criterion (Sec 13.1)
- Characteristic Equation Requirements for Sinusoidal Oscillation (Sec 13.1)

Insight into how a sinusoidal oscillator works



$$X_o(s) = R(s)$$

If $e(t)$ is any time domain excitation, no matter how small

$$E(s) = \frac{N_E(s)}{D_E(s)} = \frac{N_E(s)}{(s-x_1)(s-x_2)\cdots(s-x_m)}$$

$$R(s) = E(s)\,T(s)$$

$$R(s) = \frac{N(s)\,N_E(s)}{D(s)\,D_E(s)}$$

polynomial

polynomial

by partial fraction expansion

$$R(s) = \frac{a_1}{s-p_1} + \frac{a_2}{s-p_2} + \cdots + \frac{a_n}{s-p_n} + \frac{b_1}{s-x_1} + \cdots + \frac{b_m}{s-x_m}$$

$$r(t) = \mathcal{L}^{-1}(R(s)) = r_{N1}(t) + r_{N2}(t) + \cdots + r_{Nn}(t) + r_{E1}(t) + \cdots r_{Em}(t)$$

$$= a_1 e^{p_1 t} + a_2 e^{p_2 t} + \cdots + a_n e^{p_n t} + b_1 e^{x_1 t} + \cdots + b_m e^{x_m t}$$

$$p_k = \alpha_k + j\beta_k$$

consider $a_k e^{p_k t} = a_k e^{\alpha_k t} e^{j\beta_k t}$

recall $|v e^{jx}| = |v| |e^{jx}| = |v|$

$\therefore \; |a_k e^{p_k t}| = |a_k| e^{\alpha_k t}$

If a pole exists on the positive real axis, $\alpha_k > 0$

$$|a_k e^{p_k t}| \to \infty \quad \text{as} \quad t \to \infty$$

Consider again a term in $r_N(t)$

$$r_{Nk}(t) = a_k e^{\alpha_k t} e^{i\beta_k t}$$

If $\alpha_k = 0$

$$r_{Nk}(t) = a_k \left[\cos\beta_k t + j\sin\beta_k t\right]$$

- This output is "sinusoidal" and neither grows or decays with time

- $r(t)$ is always real (ie it has no complex components)

- The expression for $r_{Nk}(t)$ has a complex component (the $a_k \sin(\beta_k t)\, j$ term) and may appear to contradict the statement that $r(t)$ is real

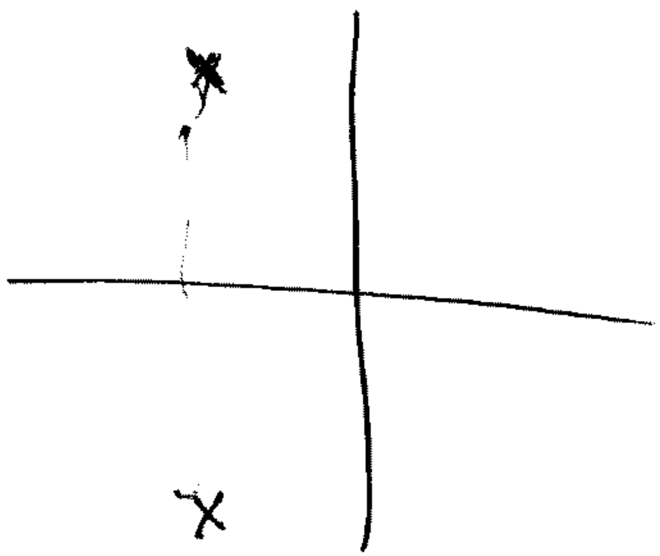
- Observe that $r(t) = \sum_{k=1}^{n} r_{Nk}(t) + \sum_{k=1}^{m} r_{Ek}(t)$

- ! the sum could still be real if the complex conjugate of $r_{Nk}(t)$ also appears in the sum

- This indeed happens and is provided for with the following theorem

Theorem: Any linear network that has a pole (or zero) at $\alpha_K + j\beta_K$ also has a pole (or zero) at $\alpha_K - j\beta_K$ whenever $\beta_K \neq 0$.

i.e. poles (and zeros) always occur in complex conjugate pairs.



- Consider any pole with a nonzero imaginary part, say $p_k = \alpha_k + j\beta_k$

- By the theorem, $p_k^* = \alpha_k - j\beta_k$ is also a pole

- Consider the component in the output due to p_k

$$r_{Nk} = a_k e^{\alpha_k} e^{j\beta_k t} = \left(a_k \cos(\beta_k t) + j\, a_k \sin(\beta_k t)\right) e^{\alpha_k t}$$

- It can be shown that the component in the output due to p_k^* is

$$r_{Nk}^* = a_k e^{\alpha_k} e^{-j\beta_k t} = \left(a_k \cos(\beta_k t) - j\, a_k \sin(\beta_k t)\right) e^{\alpha_k t}$$

- Thus both r_{Nk} and r_{Nk}^* have a complex part but their sum is <u>real</u>

$$r_{Nk} + r_{Nk}^* = \left(2 a_k \cos(\beta_k t)\right) e^{\alpha_k t}$$

- The term $r_{NK} + r_{NK}^*$ will decay to 0 if $\alpha_K < 0$ (i.e. pole in the LHP)

- The term $r_{NK} + r_{NK}^*$ will diverge to $\pm\infty$ if $\alpha_K > 0$ (i.e. pole in the RHP)

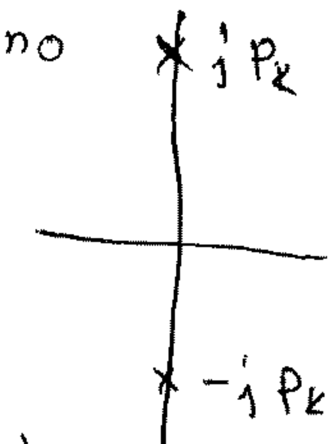
- The term $r_{NK} + r_{NK}^*$ will neither grow or decay in amplitude if $\alpha_K = 0$

The strategy for obtaining sinusoidal oscillation can now be developed

Consider a network with a single complex conjugate pole pair on the imaginary axis and no RHP poles

$$p_k = j\beta_k$$

$$p_k^* = -j\beta_k$$



The response due to any excitation (no matter how small)

$$r(t) = a_k e^{j\beta_k t} + a_k e^{-j\beta_k t}$$

$$r(t) = a_k\left(e^{j\beta_k t} + e^{-j\beta_k t}\right) \qquad \text{Recall } \frac{e^{jx}+e^{-jx}}{2} = \cos x$$

$$\therefore \quad r(t) = 2a_k \cos(\beta_k t)$$

A Criteria for Sinusoidal Oscillation

A network that has a single cc pole pair on the imaginary axis at $\pm j\omega$ and no RHP poles will have a sinusoidal output of the form

$$V_o(t) = A \sin(\omega t + \theta)$$

A & θ can not be determined from the properties of the linear network